



St. Antony's Catholic Primary School

Mathematics Policy

School Mission Statement

'God is at the heart of our school.'

With the help of our community, we will strive to learn, love and achieve.

Policy Date: Jan 22

Review Date: July 22

Progression in Calculation

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of 'real' life. Games and songs are a useful way to introduce key vocabulary involved in the four mathematical operations.

Addition Early Years

Useful guidance, models and images.

- Numicon shapes are introduced straight away and should be used to:
 - Identify 1 more/less
 - Combine pieces to add
 - Find number bonds
 - Add without counting



Children can record this by printing or drawing around Numicon pieces.

- Tens frames can also be used to:
 - identify 1 more/less
 - Find number bonds



- Children can begin to combine groups of objects using concrete apparatus:



- Construct number sentences verbally or using cards to go with practical activities.



1	+	1	=	2
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Key language which should be used:

Plus, estimate, add, more, and, sum, total, make, altogether, score, double.

One more, two more, three more...

How many more make...?

How many more is...than...?

Same as

- Children should be encouraged to read number sentences aloud in different ways.

$$3 + 2 = 5$$

"Three add two equals 5", "5 is equal to three and two" or

"5 is the same as three and two".

- Children make a record in pictures, words or symbols of addition activities.



$$3 + 5 = 8$$

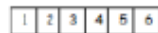
- Solve simple problems using fingers.



$$5 + 1 = 6$$

- Introduce number tracks to count-up on and to find one more:

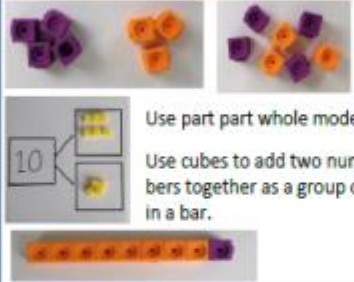
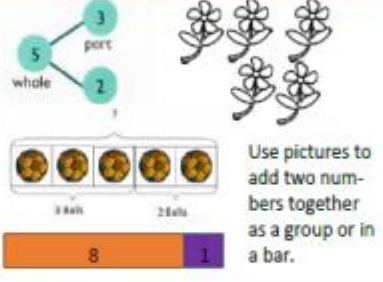
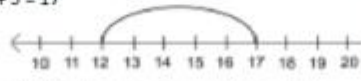
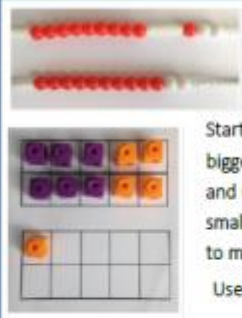
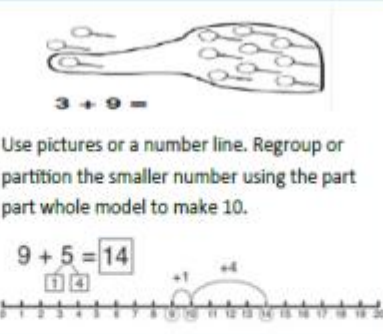
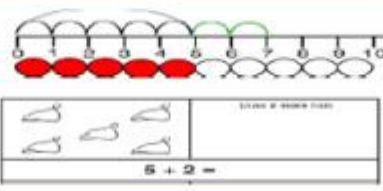
What is one more than 4?



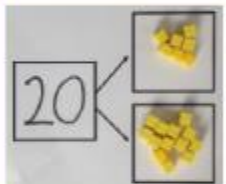
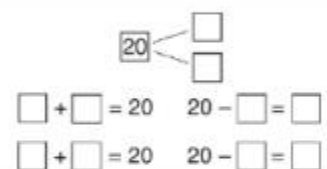
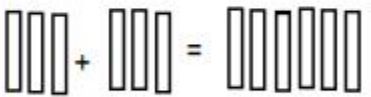
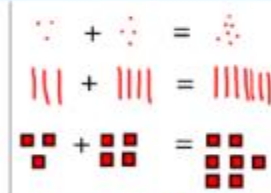
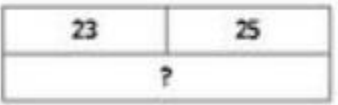
- Use number lines alongside number tracks and practical apparatus to solve addition calculations and word problems.



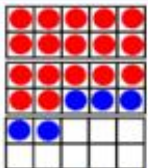
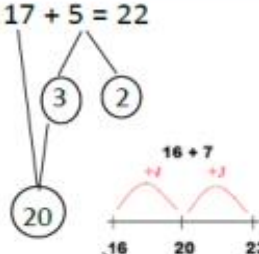
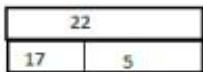
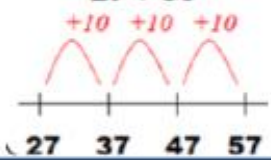
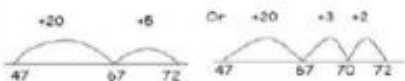
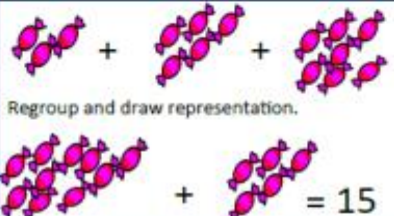
Children will need the opportunity to look at and talk about different models and images as they move between different representations.

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$  Use the part-part whole diagram as shown above to move into the abstract. $10 = 6 + 4$
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 Start with the bigger number and use the smaller number to make 10. Use ten frames.	$3 + 9 =$  Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 5 + 2 =	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

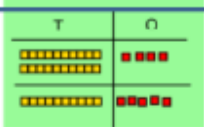
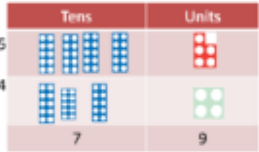
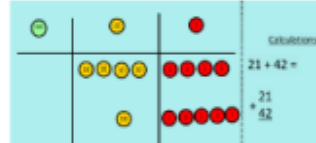
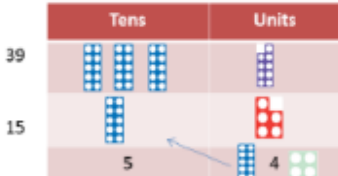
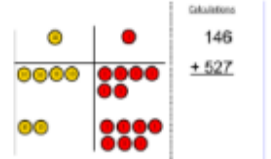
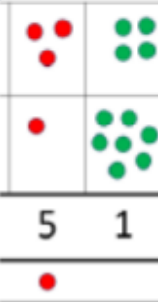
Y1
+

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$ $30 + 50 = \underline{\hspace{2cm}}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\square\square + \square\square = \square\square\square\square$ 	 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

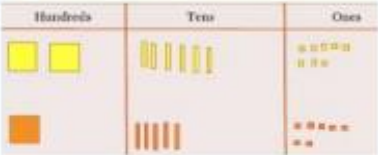
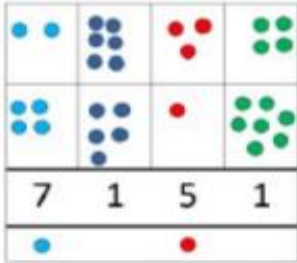
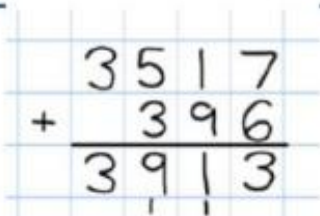
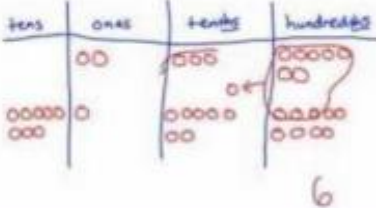
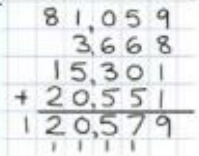
Y2
+

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten' Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 - 17 = 5 22 - 5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	27 + 30 	27 + 10 = 37 27 + 20 = 47 27 + 30 = 57
Add two 2-digit numbers	 Model using dienes, place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 = 25 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit	 Regroup and draw representation. 4 + 7 + 6 = 17	4 + 7 + 6 = 10 + 7 10 + 7 = 17 Combine the two numbers that make/ bridge ten then add on the third.

Y2
+

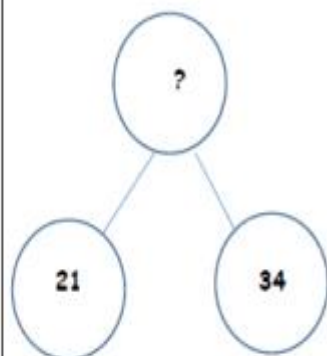
Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$

Y3
+

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.	2.37 + 81.79 	72.8 + 54.6 127.4 11 £ 23.59 + £ 7.55 £ 31.14
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	 Insert zeros for place holders. 

Y4
Y5
Y6
+

Fluency variation, different ways to ask children to solve addition calculations e.g. $21 + 34$



Sam saved £21 one week and £34 another. How much did he save in total?

$21 + 34 = 55$. Prove it!

(Reasoning but the children need to be fluent in representing this)

21

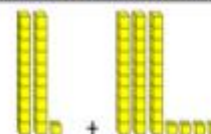
+34

—

$21 + 34 =$

$= 21 + 34$

What's the sum of twenty one and thirty four?



Always use missing digit problems too:

Tens	Ones
	?
?	5

Subtraction Early Years

Useful guidance, models and images.

- Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left. E.g. $5 - 2 =$



- Construct number sentences verbally or using cards to go with practical activities.

● ● ● ● ● ✕
 $5 - 1 = 4$

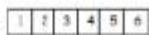
- Children should be encouraged to read sentences aloud in different ways... "five subtract one leaves four", "four is equal to five subtract one" or "four is the same as five subtract one".
- Children make a record in pictures, words or symbols of subtraction activities.



- Solve simple problems using fingers.



- Introduce number tracks to children to find one less: What is 1 less than 6?



Key language which should be used:

Take (away), estimate, leave, left, fewer, less, difference between, the same as, counting/hopping back.

How many are left/left over?

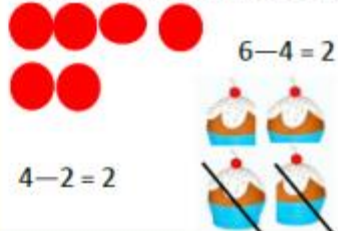
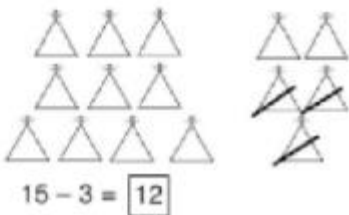
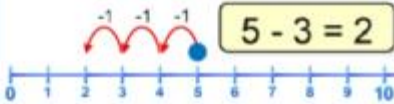
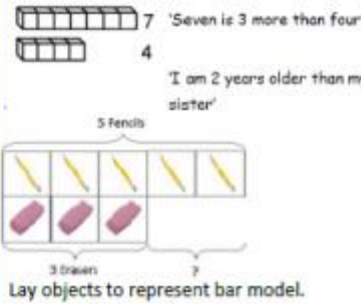
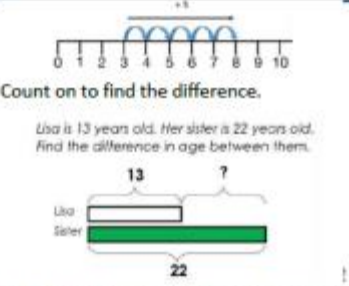
How many have gone - one less, two less, three less...

How many fewer is...?

- Number lines can be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back showing hops back on the number line.

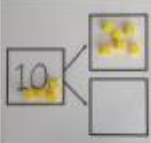
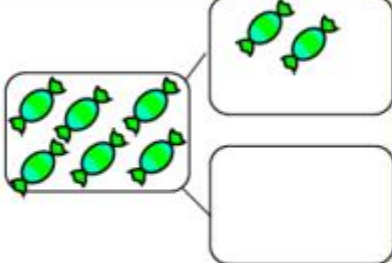
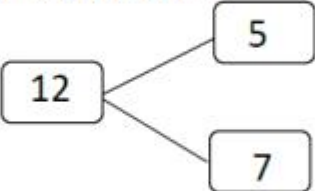
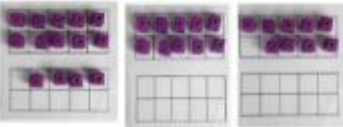


Children will need the opportunity to look at and talk about different models and images as they move between different representations.

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.  $6 - 4 = 2$ $4 - 2 = 2$	 $15 - 3 = 12$ Cross out drawn objects to show what has been taken away.	$7 - 4 = 3$ $16 - 9 = 7$
Counting back	 Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister' 5 pencils 3 brown Lay objects to represent bar model.	 Count on to find the difference. Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them. 13 22 Draw bars to find the difference between 2 numbers.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

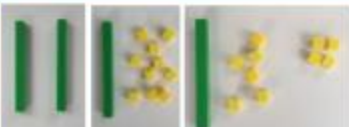
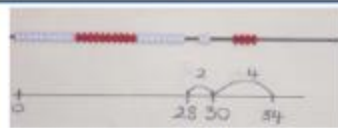
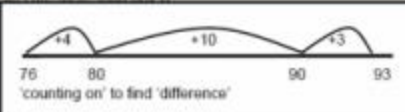
Y1

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Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what's the other part? $10 - 6 = 4$	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 $5 - 2 = 3$		 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$

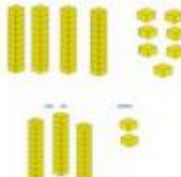
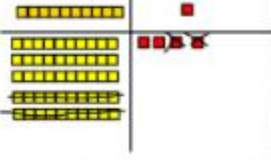
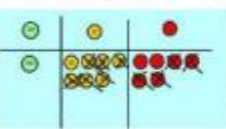
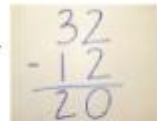
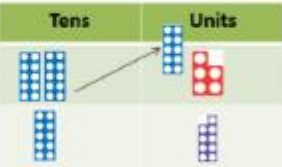
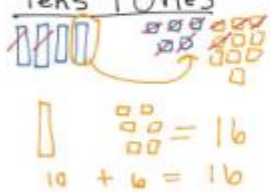
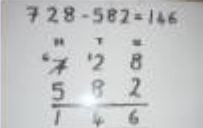
Y1

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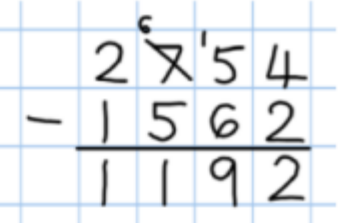
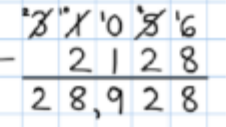
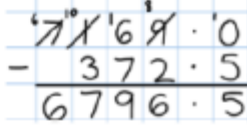
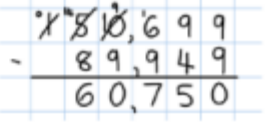
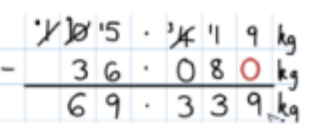
Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 $20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	$93 - 76 = 17$

Y2

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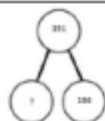
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	Use base 10 or Numicon to model  $47 - 32$ Show how you partition numbers to subtract. Again make the larger number first. 	 Draw representations to support understanding  Calculations $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.	$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$ Tens Ones  Children may draw base ten or PV counters and cross off.	$836 - 254 = 582$  Begin by partitioning into pv columns $728 - 582 = 146$  Then move to formal method.

Y3
-

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. <i>Introduce decimal subtraction through context of money</i>	$234 - 179$  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. <i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i>	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

Y4
Y5
Y6
-

Fluency variation, different ways to ask children to solve addition calculations e.g. $21 + 34$



Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

$$391 - 186$$

$$\square = 391 - 186$$

391

-186

Find the difference between 391 and 186.

Subtract 186 from 391.

What is 186 less than 391?

What's the calculation? What's the answer?



$$\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$$

Multiplication Early Years

Useful guidance, models and images.

- The link between addition and multiplication can be introduced through doubling.
- Numicon can be used to visualise the repeated adding of the same number.



Children can record this by printing or drawing around the Numicon pieces.

- Begin with mostly concrete or pictorial representations.



e.g. How many groups of 2 are there? $2 + 2 + 2 + 2 + 2$, so 5 groups of 2

- Use 'real' life contexts and use of practical equipment to count in repeated groups of the same size.



How many wheels are there altogether?



How much money do I have?

- Count in twos, fives, tens both aloud and with objects.



Key language which should be used:

Lots of, groups of, times, multiply, multiplied by, multiple of.

Once, twice, three times...

...times as (big, long, wide...)

repeated addition

double

estimate

add again and again

- Give children multiplication problems set in a 'real' life context. Encourage them to visualise the problem using concrete materials or by drawing pictures.
e.g. How many fingers on two hands?



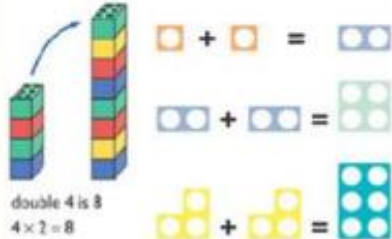
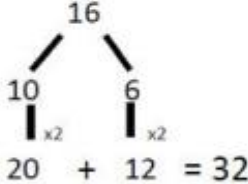
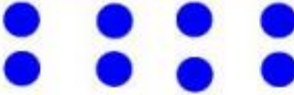
How many sides on three triangles?



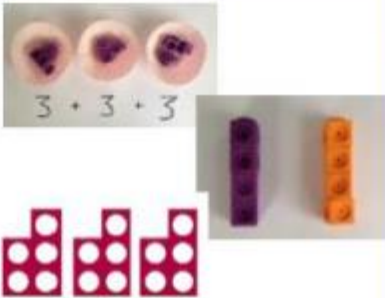
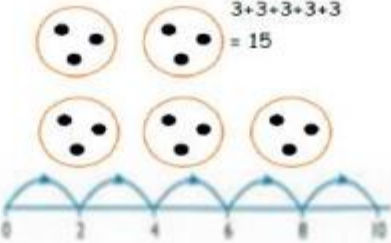
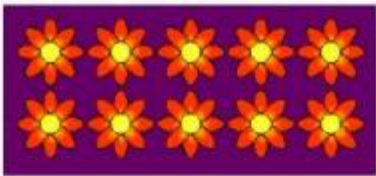
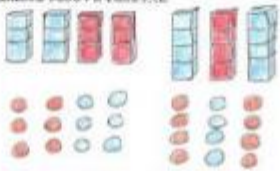
How many legs on four ducks?



- Children should be encouraged to read number sentences aloud in different ways... "five times two makes ten", "ten is equal to five multiplied by two" or "ten is the same as five lots of two".

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together.  16 10 6 x2 x2 20 + 12 = 32
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	  $\square \times \square = 8$ Use manipulatives to create equal groups.	Draw  to show $2 \times 3 = 6$ Draw and make representations	$2 \times 4 = 8$

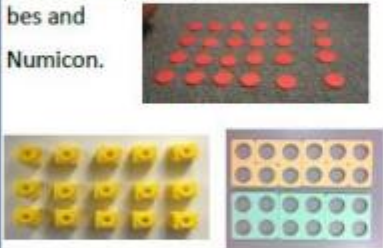
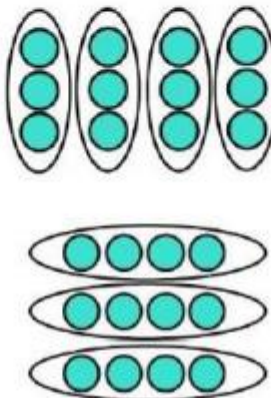
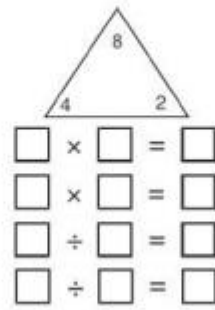
Y1
X

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding. 	$3 \times 2 = 6$ $2 \times 5 = 10$

Y1
x

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters. $40 + 12 = 52$	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. $16 \begin{matrix} \swarrow & \searrow \\ 10 & 6 \end{matrix} \begin{matrix} \downarrow \times 2 & \downarrow \times 2 \\ 20 & 12 \end{matrix} \rightarrow 20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$

Y2
X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity. 	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse <i>This should be taught alongside division, so pupils learn how they work alongside each other.</i>			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

Y2
x

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method. x 10 3 </		

Y3
x

Objective & Strategy	Concrete	Pictorial	Abstract																																															
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Fill each row with 126 Add up each col. making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. <table><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> 210 + 35 = 245	x	30	5	7	210	35																																									
x	30	5																																																
7	210	35																																																
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 <table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr></table> It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	Hundreds	Tens	Ones	3	2	1	3	2	1	3	2	1	3	2	1	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> The grid method may be used to show how this relates to a formal written method.	x	300	20	7	4	1200	80	28	<table><tr><td>327</td></tr><tr><td>x 4</td></tr><tr><td>28</td></tr><tr><td>80</td></tr><tr><td>1200</td></tr><tr><td>1308</td></tr></table> This may lead to a compact method. <table><tr><td></td><td>3</td><td>2</td><td>7</td></tr><tr><td>x</td><td></td><td></td><td>4</td></tr><tr><td></td><td>1</td><td>3</td><td>0</td><td>8</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td></td></tr></table>	327	x 4	28	80	1200	1308		3	2	7	x			4		1	3	0	8			1	2	
Hundreds	Tens	Ones																																																
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Y4
x

Objective & Strategy	Concrete	Pictorial	Abstract																																															
Column Multiplication for 3 and 4 digits x 1 digit.	<table border="1"> <thead> <tr> <th>Hundreds</th><th>Tens</th><th>Ones</th></tr> </thead> <tbody> <tr> <td></td><td></td><td></td></tr> <tr> <td></td><td></td><td></td></tr> <tr> <td></td><td></td><td></td></tr> <tr> <td></td><td></td><td></td></tr> </tbody> </table> It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$	Hundreds	Tens	Ones													<table border="1"> <tr> <td>x</td><td>300</td><td>20</td><td>7</td></tr> <tr> <td>4</td><td>1200</td><td>80</td><td>28</td></tr> </table>	x	300	20	7	4	1200	80	28	$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$ <table border="1"> <tr><td>3</td><td>2</td><td>7</td></tr> <tr><td>x</td><td></td><td>4</td></tr> <tr><td>1</td><td>3</td><td>0</td></tr> <tr><td></td><td>2</td><td></td></tr> </table> This will lead to a compact method.	3	2	7	x		4	1	3	0		2													
Hundreds	Tens	Ones																																																
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1	3	0																																																
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Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	<table border="1"> <tr> <td></td><td>10</td><td>8</td></tr> <tr> <td>10</td><td>100</td><td>80</td></tr> <tr> <td>3</td><td>30</td><td>24</td></tr> </table> Continue to use bar modelling to support problem solving		10	8	10	100	80	3	30	24	<table border="1"> <tr><td></td><td>1</td><td>8</td></tr> <tr><td>x</td><td>1</td><td>3</td></tr> <tr><td></td><td>5</td><td>4</td></tr> <tr><td></td><td>2</td><td></td></tr> <tr><td>1</td><td>8</td><td>0</td></tr> <tr><td>2</td><td>3</td><td>4</td></tr> </table> 18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first <table border="1"> <tr><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>x</td><td></td><td></td><td>6</td></tr> <tr><td>7</td><td>4</td><td>0</td><td>4</td></tr> <tr><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>1</td><td>9</td><td>7</td><td>4</td></tr> </table> (1234 x 6) (1234 x 10)		1	8	x	1	3		5	4		2		1	8	0	2	3	4	1	2	3	4	x			6	7	4	0	4	1	2	3	4	1	9	7	4
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7	4	0	4																																															
1	2	3	4																																															
1	9	7	4																																															

Y5
Y6
x

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. $ \begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array} $

Y6
X

Fluency variation, different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

With the counters, prove that
 $6 \times 23 = 138$

Why is $6 \times 23 = 32 \times 6$?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23

$$6 \times 23 = \square$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

What's the calculation?

What's the answer?



Division Early Years

Useful guidance, models and images.

- Solve problems including doubling, halving and sharing.
- Show children representations of division as grouping and sharing.
- Introduce through halving.
e.g. Concrete and pictorial representations linked to 'real' life.



- Grouping
Mum has 6 socks. She grouped them into pairs - how many pairs did she make? How many socks did she have altogether?



- Sharing - this is a useful way of introducing young children to fractions and calculating with fractions.
e.g. I have ten sweets. I want to share them with my friend. How many will we each have?



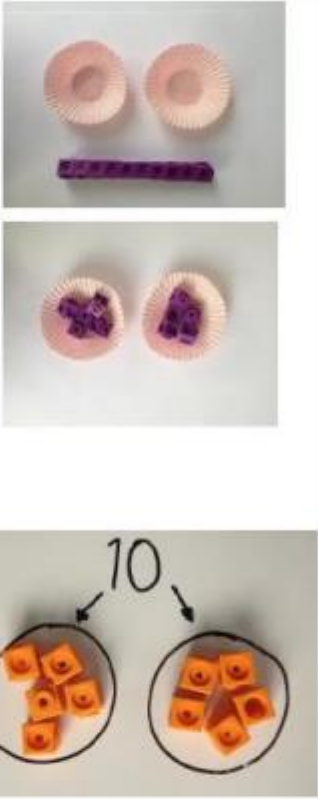
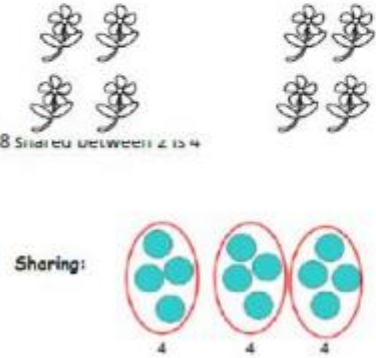
I have got a whole pizza to share between two people. Can you cut the pizza in half?



- Children can record in pictures, words or symbols of division activities.

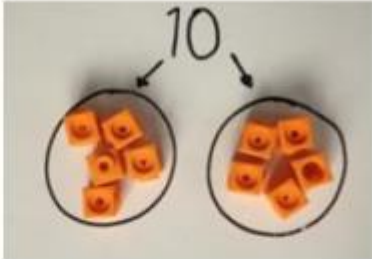
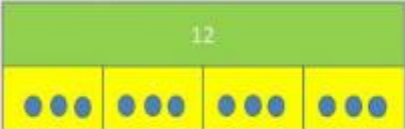
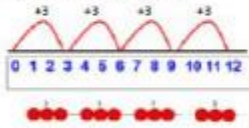
Key language which should be used:

Halve, share, share equally,
one each, two each...
group in pairs, threes,
equal groups of
divide
divided by
divided into
left over
estimate
fraction
half
halves
whole
quarter

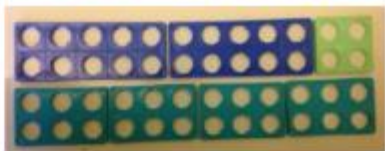
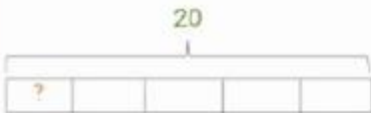
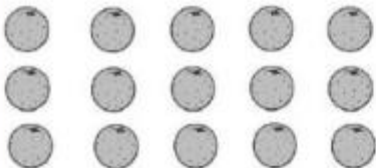
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing <i>Use Gordon ITPs for modelling</i>	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 Shared equally between 2 is 4 Sharing: 12 shared between 3 is 4	12 shared between 3 is 4

Y1

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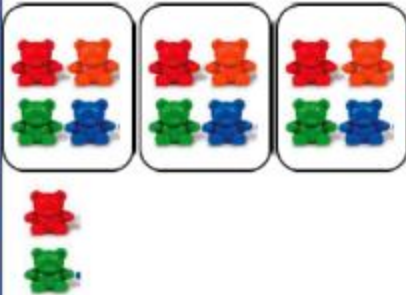
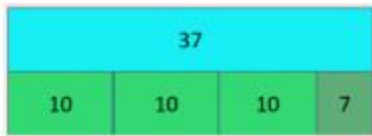
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children use bar modelling to show and support understanding.  $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  $12 \div 3 = 4$ Think of this as a multiplication. What is the number of groups you are dividing by and work out how many would be within each group.  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Y2
÷

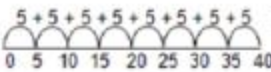
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 $96 \div 3 = 32$ 	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Y3

÷

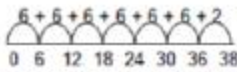
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	$14 \div 3 =$ Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show the remainder using r. $\begin{array}{ccccccc} 29 \div 8 = 3 \text{ REMAINDER } 5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \quad \uparrow \\ \text{dividend} \quad \text{divisor} \quad \text{quotient} \quad \text{remainder} \end{array}$

Example without remainder:
 $40 \div 5$
 Ask "How many 5s in 40?"



$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 \text{ fives}$

Example with remainder:
 $38 \div 6$

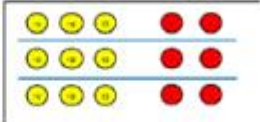
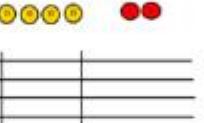
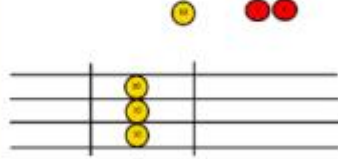
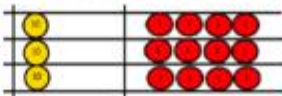
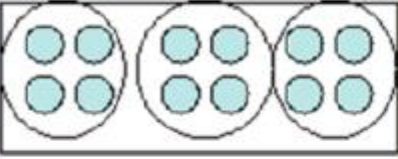


$6 + 6 + 6 + 6 + 6 + 6 + 2 = 6 \text{ sixes with a remainder of } 2$

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Y3

÷

Objective & Strategy	Concrete	Pictorial	Abstract				
Divide at least 3 digit numbers by 1 digit. Short Division	$96 \div 3$ <table><thead><tr><th>Tens</th><th>Units</th></tr></thead><tbody><tr><td>3</td><td>2</td></tr></tbody></table>  Use place value counters to divide using the bus stop method alongside  $42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Tens	Units	3	2	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 258} \end{array}$ Finally move into decimal places to divide the total accurately. $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$
Tens	Units						
3	2						

Y4
Y5
Y6
÷

Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y6
long
÷

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \end{array}$ Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 1 \end{array}$ To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \downarrow \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \downarrow \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \downarrow \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ The division is over since there are no more digits in the dividend. The quotient is 29.

Y6
long
÷

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{2} \end{array}$ Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \end{array}$ Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \end{array}$ Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ Divide 2 into 7. Place 3 into the quotient.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \end{array}$ Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \end{array}$ Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \end{array}$ There are no more digits to drop down. The quotient is 139.

Y6
long
÷

Fluency variation, different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



(Use practical equipment and share out each partitioned number).

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

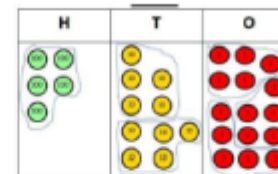
$$5 \overline{)615}$$

$$615 \div 5 \quad \square =$$

$$\square = 615 \div 5$$

How many 5's go into 615?

What's the calculation? What's the answer?



Key Vocabulary

Addition

EYFS	add, one more, count, more, plus, and, make, altogether, total, equal to, same as, double, most, count on, number line , part whole
Y1	Use and apply language from the Early Stage in a range of contexts
Y2	sum, tens, ones, partition, addition, column, tens boundary
Y3	hundreds boundary, increase, vertical, 'carry', expanded, compact
Y4	thousands, hundreds, digits, inverse
Y5	decimal places, decimal point, tenths, hundredths, thousandths
Y6	Use and apply language from stage five in a range of contexts.

Subtraction

EYFS	Take, take away, less, minus, subtract, leaves, most, least, count back , how many left? part, whole
Y1	Equal to, distance between, how many more, how many fewer / less than, how much less is?
Y2	difference, count on, strategy, partition, tens, ones
Y3	exchange, decrease, hundreds, value, digit
Y4	inverse
Y5	tenths, hundredths, decimal point, decimal
Y6	Use and apply language from stage five in a range of contexts.

Multiplication

EYFS	double, count, altogether, two groups, twice, the amount same again, repeat, more, patter
Y1	groups of, lots of, times, array, altogether, multiply, count
Y2	multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times
Y3	partition, grid method, multiple, product, tens, ones, value
Y4	inverse
Y5	square, factor, integer, decimal, short/long multiplication, 'carry
Y6	tenths, hundredths, decimal

Division

EYFS	share, share equally, one each, two each..., half, split, same, fair, even, halving, sharing
Y1	share, share equally, one each, two each..., group, groups of, lots of, array
Y2	divide, divided by, divided into, division, grouping, number line, left, left over
Y3	inverse, short division, 'carry', remainder, multiple
Y4	divisible by, factor
Y5	quotient, prime number, prime factors, composite number (non-prime)
Y6	common factor

Other agreed vocabulary:

- a) Do not use "equals" but "is equal to" or "is the same as"
- b) The terms "parts" and "whole" should be introduced with addition and subtraction to support transition from concrete/visual strategies to abstract
- c) "ones" not "units" as this better relates to diene values and can be confused with measurement
- d) "zero" not "oh/nought"
- e) "digit" rather than "number" when discussing place value. E.g. in '23', 2 is a digit worth 2 tens or 20

Times tables expectations

'By the end of year 4, pupils should have memo-rised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.'

National Curriculum, 2014

The minimum requirements for Year groups are as follows:

Year 1 – counting in 2s, 5s and 10s

Year 2 – recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

Year 3 – recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Year 4 – recall multiplication and division facts for multiplication tables up to 12×12

Year 5&6 – apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.